PAST EXAM QUESTIONS

This section contains the Subject CM1 exam questions from April 2014 to April 2024 that are related to the topics covered in this booklet.

Solutions are given later in this booklet. These give enough information for you to check your answer, including working, and also show you what an adequate examination answer should look like. Further information may be available in the Examiners' Report, ASET or Course Notes. (ASET can be ordered from ActEd.)

We first provide you with a list of exam questions in cross-reference grid of key words or phrases that indicate the main subject areas of each exam question. You can use this, if you wish, to select the questions that relate just to those aspects that you may be particularly interested in reviewing. Alternatively, you can choose to ignore the grid, and instead attempt the questions without having any clues as to their content.

		Basic interest				Variable force of interest			Annuities	
Question	**Tick when attempted**	Accumulation/ discount	Converting	Treasury bill	Nominal interest/ discount	Accumulation/ discount	General A(t) / v(t) expression	Payment streams	Level annuities	Increasing annuities
1			✓		✓					
2			✓			✓		(✓)		
3		✓	(✓)	✓						
4									✓	✓
5			✓			✓		✓		
6		✓								
7				✓						
8		✓			✓				✓	

		Basic interest				Variable force of interest			Annuities	
Question	**Tick when attempted**	Accumulation/ discount	Converting	Treasury bill	Nominal interest/ discount	Accumulation/ discount	General A(t) / v(t) expression	Payment streams	Level annuities	Increasing annuities
9						✓	✓	✓		
10		✓		✓						
11			✓		✓					
12						✓		✓		
13					✓	✓				
14			✓		✓					
15					✓				✓	
16				✓						
17						✓		✓		
18			✓		✓					
19		✓	✓							
20				✓						
21									✓	
22					✓	✓		✓		
23		✓	✓		✓				✓	
24			✓		✓	✓	✓	✓		
25		✓		✓						
26			✓		✓					
27						✓		✓		
28			✓		✓					

		Basic interest				Variable force of interest			Annuities	
Question	**Tick when attempted**	Accumulation/ discount	Converting	Treasury bill	Nominal interest/ discount	Accumulation/ discount	General A(t) / v(t) expression	Payment streams	Level annuities	Increasing annuities
29						✓		✓		
30						✓		✓		
31		✓	✓		✓				✓	
32			✓		✓	✓	✓			
33						✓	✓			
34		✓	✓							
35		✓							✓	
36						✓	✓			
37		✓			✓				✓	
38						✓	✓	✓		

1 Subject CT1, April 2014, Question 3

£900 accumulates to £925 in four months.

Calculate the following:

(i) the nominal rate of interest per annum convertible half-yearly [2]

(ii) the nominal rate of discount per annum convertible quarterly [2]

(iii) the simple rate of interest per annum. [2]
[Total 6]

2 Subject CT1, April 2014, Question 11

An individual can obtain a force of interest per annum at time t, measured in years, as given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.01t & 0 \le t < 4 \\ 0.07 & 4 \le t < 6 \\ 0.09 & 6 \le t \end{cases}$$

(i) Calculate the amount the individual would need to invest at time $t = 0$ in order to receive a continuous payment stream of \$3,000 per annum from time $t = 4$ to $t = 10$. [6]

(ii) Calculate the equivalent constant annual effective rate of interest earned by the individual in part (i). [3]

[Total 9]

3 Subject CT1, September 2014, Question 3

A 91-day treasury bill is bought for £98.83 and is redeemed at £100.

(i) Calculate the annual effective rate of interest from the bill. [3]

(ii) Calculate the annual equivalent simple rate of interest. [2]

[Total 5]

4 Subject CT1, September 2014, Question 5

Calculate, at a rate of interest of 5% per annum effective:

(i) $a^{(12)}_{\overline{5}|}$ [1]

(ii) ${}_{4|}a_{\overline{15}|}$ [1]

(iii) $(I\bar{a})_{\overline{10}|}$ [1]

(iv) $(\bar{I}\bar{a})_{\overline{10}|}$ [1]

(v) the present value of an annuity that is paid annually in advance for 10 years with a payment of 12 in the first year, 11 in the second year and thereafter reducing by 1 each year. [2]

[Total 6]

5 Subject CT1, September 2014, Question 7

The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 & \text{for} \quad 0 < t \le 10 \\ 0.003t & \text{for} \quad 10 < t \le 20 \\ 0.0001t^2 & \text{for} \quad t > 20 \end{cases}$$

(i) Calculate the present value of a unit sum of money due at time $t = 28$. [7]

(ii) (a) Calculate the equivalent constant force of interest from $t = 0$ to $t = 28$.

(b) Calculate the equivalent annual effective rate of discount from $t = 0$ to $t = 28$. [3]

A continuous payment stream is paid at the rate of $e^{-0.04t}$ per unit time between $t = 3$ and $t = 7$.

(iii) Calculate the present value of the payment stream. [4]
[Total 14]

6 Subject CT1, April 2015, Question 2

Calculate the time in days for £3,000 to accumulate to £3,800 at:

(a) a simple rate of interest of 4% per annum.

(b) a compound rate of interest of 4% per annum effective. [4]

7 Subject CT1, April 2015, Question 3

A 182-day treasury bill, redeemable at $100, was purchased for $96.50 at the time of issue and later sold to another investor for $98 who held the bill to maturity. The rate of return received by the initial purchaser was 4% per annum effective.

(i) Calculate the length of time in days for which the initial purchaser held the bill. [2]

(ii) Calculate the annual simple rate of return achieved by the second investor. [2]

(iii) Calculate the annual effective rate of return achieved by the second investor. [2]

[Total 6]

8 Subject CT1, April 2015, Question 5

An investor pays £120 per annum into a savings account for 12 years. In the first four years, the payments are made annually in advance. In the second four years, the payments are made quarterly in advance. In the final four years, the payments are made monthly in advance.

The investor achieves a yield of 6% per annum convertible half-yearly on the investment.

Calculate the accumulated amount in the savings account at the end of 12 years. [7]

9 Subject CT1, April 2015, Question 10

The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is given by

$$\delta(t)=\begin{cases}0.08 & \text{for } 0\le t\le 4\\ 0.12-0.01t & \text{for } 4<t\le 9\\ 0.05 & \text{for } t>9\end{cases}$$

(i) Determine the discount factor, $v(t)$, that applies at time t for all $t\ge 0$. [5]

(ii) Calculate the present value at $t=0$ of a payment stream, paid continuously from $t=10$ to $t=12$, under which the rate of payment at time t is $100e^{0.03t}$. [4]

(iii) Calculate the present value of an annuity of £1,000 paid at the end of each year for the first three years. [3]

[Total 12]

10 Subject CT1, September 2015, Question 1

An investor wishes to obtain a rate of interest of 3% per annum effective from a 91-day treasury bill.

Calculate:

(a) the price that the investor must pay per £100 nominal.

(b) the annual simple rate of discount from the treasury bill. [3]

11 Subject CT1, September 2015, Question 2

The nominal rate of discount per annum convertible monthly is 5.5%.

(i) Calculate, giving all your answers as a percentage to three decimal places:

(a) the equivalent force of interest.

(b) the equivalent effective rate of interest per annum.

(c) the equivalent nominal rate of interest per annum convertible monthly. [3]

(ii) Explain why the nominal rate of interest per annum convertible monthly calculated in part (i)(c) is less than the equivalent annual effective rate of interest calculated in part (i)(b). [1]

(iii) Calculate, as a percentage to three decimal places, the effective annual rate of discount offered by an investment that pays £159 in eight years' time in return for £100 invested now. [1]

(iv) Calculate, as a percentage to three decimal places, the effective annual rate of interest from an investment that pays 12% interest at the end of each two-year period. [1]
[Total 6]

12 Subject CT1, September 2015, Question 5

An individual can obtain a force of interest per annum at time t , measured in years, as given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.005t & 0 \le t \le 3 \\ 0.005 & t > 3 \end{cases}$$

(i) Determine the amount the individual would need to invest at time $t = 0$ in order to receive a continuous payment stream of £5,000 per annum from time $t = 3$ to time $t = 6$. [5]

(ii) Determine the equivalent constant annual effective rate of interest earned by the individual in part (i). [3]

(iii) Determine the amount an individual would accumulate from the investment of £300 from time $t = 0$ to time $t = 50$. [2]

[Total 10]

13 Subject CT1, April 2016, Question 6

The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.06 & 0 \le t \le 4 \\ 0.10 - 0.01t & 4 < t \le 7 \\ 0.01t - 0.04 & 7 < t \end{cases}$$

(i) Calculate, showing all working, the value at time $t = 5$ of £10,000 due for payment at time $t = 10$. [5]

(ii) Calculate the constant rate of discount per annum convertible monthly which leads to the same result as in part (i). [2]

[Total 7]

14 Subject CT1, September 2016, Question 1

The nominal rate of interest per annum convertible quarterly is 5%.

Calculate, giving all the answers as a percentage to three decimal places:

(i) the equivalent annual force of interest. [1]

(ii) the equivalent effective rate of interest per annum. [1]

(iii) the equivalent nominal rate of discount per annum convertible monthly. [2]

[Total 4]

15 Subject CT1, September 2016, Question 2

The nominal rate of interest per annum convertible quarterly is 2%.

Calculate the present value of a payment stream paid at a rate of €100 per annum, monthly in advance for 12 years. [4]

16 Subject CT1, September 2016, Question 6

At the beginning of 2015 a 182-day commercial bill, redeemable at £100, was purchased for £96 at the time of issue and later sold to a second investor for £97.50. The initial purchaser obtained a simple rate of interest of 3.5% per annum before selling the bill.

(i) Calculate the annual simple rate of return which the initial purchaser would have received if they had held the bill to maturity. [2]

(ii) Calculate the length of time in days for which the initial purchaser held the bill. [2]

The second investor held the bill to maturity.

(iii) Calculate the annual effective rate of return achieved by the second investor. [2]

[Total 6]

17 Subject CT1, September 2016, Question 12

The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is given by:

$$\delta(t) = \begin{cases} 0.03 & \text{for} \quad 0 \le t \le 10 \\ at & \text{for} \quad 10 < t \le 20 \\ bt & \text{for} \quad t > 20 \end{cases}$$

where a and b are constants.

The present value of £100 due at time 20 is 50.

(i) Calculate a. [5]

The present value of £100 due at time 28 is 40.

(ii) Calculate b. [4]

(iii) Calculate the equivalent annual effective rate of discount from time 0 to time 28. [2]

A continuous payment stream is paid at the rate of $e^{-0.04t}$ per annum between $t = 3$ and $t = 7$.

(iv) (a) Calculate, showing all workings, the present value of the payment stream.

(b) Determine the level continuous payment stream per annum from time $t=3$ to time $t=7$ that would provide the same present value as the answer in part (iv)(a) above. [8]

[Total 19]

18 Subject CT1, April 2017, Question 1

Calculate the nominal rate of discount per annum convertible monthly which is equivalent to:

(i) an effective rate of interest of 1% per quarter. [2]

(ii) a force of interest of 5% per annum. [2]

(iii) a nominal rate of discount of 4% per annum convertible every three months. [2]

[Total 6]

19 Subject CT1, September 2017, Question 1

(i) Calculate the time in days for £6,000 to accumulate to £7,600 at:

(a) a simple rate of interest of 3% per annum.

(b) a compound rate of interest of 3% per annum effective.

(c) a force of interest of 3% per annum. [6]

Note: You should assume there are 365 days in a year.

(ii) Calculate the effective rate of interest per half-year which is equivalent to a force of interest of 3% per annum. [1]

[Total 7]

20 Subject CT1, September 2017, Question 3

An investor is considering two investments. One is a 91-day deposit which pays a compound rate of interest of 3% per annum effective. The second is a government bill.

Calculate the annual simple rate of discount from the government bill if both investments are to provide the same effective rate of return. [3]

21 Subject CT1, September 2017, Question 6

An investor has a choice of two 15-year savings plans, A and B, issued by a company. In both plans, the investor pays contributions of \$100 at the start of each month and the contributions accumulate at an effective rate of interest of 4% per annum before any allowance is made for expenses.

In plan A, the company charges for expenses by deducting 1% from the annual effective rate of return.

In plan B, the company charges for expenses by deducting \$15 from each of the first year's monthly contributions before they are invested. In addition it deducts 0.3% from the annual effective rate of return.

Calculate the percentage by which the accumulated amount in Plan B is greater than the accumulated amount in Plan A, at the end of the 15 years. [6]

22 Subject CT1, September 2017, Question 9

The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.09 - 0.003t & 0 \le t \le 10 \\ 0.06 & t > 10 \end{cases}$$

(i) Calculate the corresponding constant effective annual rate of interest for the period from $t = 0$ to $t = 10$. [4]

(ii) Express the rate of interest in part (i) as a nominal rate of discount per annum convertible half-yearly. [1]

(iii) Calculate the accumulation at time $t = 15$ of £1,500 invested at time $t = 5$. [3]

(iv) Calculate the corresponding constant effective annual rate of discount for the period $t = 5$ to $t = 15$. [1]

(v) Calculate the present value at time $t = 0$ of a continuous payment stream payable at a rate of $10e^{0.01t}$ from time $t = 11$ to time $t = 15$. [6]
[Total 15]

23 Subject CT1, April 2018, Question 3

An investor pays £80 at the start of each month into a 25-year savings plan.

The contributions accumulate at an effective rate of interest of 3% per half-year for the first 10 years, and at a force of interest of 6% per annum for the final 15 years.

Calculate the accumulated amount in the savings plan at the end of 25 years. [6]

24 Subject CT1, April 2018, Question 10

The force of interest $\delta(t)$ is a function of time, and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.24 - 0.02t & 0 < t \le 6 \\ 0.12 & 6 < t \end{cases}$$

(i) Derive, and simplify as far as possible, expressions in terms of t for the present value of a unit investment made at any time, t. You should derive separate expressions for each time interval $0 < t \le 6$ and $6 < t$. [5]

(ii) Determine the discounted value at time $t = 4$ of an investment of £1,000 due at time $t = 10$. [2]

(iii) Calculate the constant nominal annual interest rate convertible monthly implied by the transaction in part (ii). [2]

(iv) Calculate the present value of a continuous payment stream invested from time $t = 6$ to $t = 10$ at a rate of $\rho(t) = 20e^{0.36+0.32t}$ per annum.[4]

[Total 13]

25 Subject CT1, September 2018, Question 1

An investor is considering two investments. One investment is a 91-day bond issued by a bank which pays a rate of interest of 4% per annum effective. The second is a 91-day treasury bill which pays out €100.

(i) Calculate the price of the treasury bill and the annual simple rate of discount from the treasury bill if both investments are to provide the same effective rate of return. [3]

(ii) Suggest one factor, other than the rate of return, which might determine which investment is chosen. [1]

[Total 4]

26 Subject CT1, September 2018, Question 2

The effective rate of discount per annum is 5%.

Calculate:

(i) the equivalent force of interest [1]

(ii) the equivalent rate of interest per annum convertible monthly [2]

(iii) the equivalent rate of discount per annum convertible monthly. [1]

[Total 4]

27 Subject CT1, September 2018, Question 7

The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 & 0 \le t \le 10 \\ 0.003t & t > 10 \end{cases}$$

(i) Calculate the present value of a unit sum of money due at time $t = 20$. [4]

(ii) Calculate the equivalent constant force of interest from $t = 0$ to $t = 20$. [2]

(iii) Calculate the present value at time $t = 0$ of a continuous payment stream payable at a rate of $e^{-0.06t}$ from time $t = 4$ to time $t = 8$. [4]

[Total 10]

28 Subject CM1, April 2019, Question 5

Calculate, as a percentage to four decimal places, the nominal rate of interest per annum convertible half-yearly which is equivalent to:

(i) an effective rate of discount of 0.5% per month. [2]

(ii) a nominal rate of discount of 6% per annum convertible every two years. [2]

(iii) a nominal rate of interest of 6% per annum convertible quarterly. [2]

[Total 6]

29 Subject CM1, April 2019, Question 8

The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t)=\begin{cases}0.03+0.005t & 0\le t<2\\ 0.045-0.0025t & 2\le t<10\\ 0.02 & t\ge 10\end{cases}$$

(i) Calculate the accumulated amount at time $t=9$ of an investment of £15,000 made at time $t=1$. [4]

(ii) Calculate the present value at time $t=0$ of a payment stream paid continuously from time $t=10$ to time $t=12$, under which the rate of payment at time t is $\rho(t)=60e^{0.02t}$. [6]

[Total 10]

30 Subject CM1, September 2019, Question 10

The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t)=\begin{cases}0.03+0.01t & 0\le t<4\\ 0.07 & 4\le t<6\\ 0.09 & t\ge 6\end{cases}$$

(i) Calculate the accumulated amount at time $t=6$ of a lump sum of 10 units invested at time $t=0$. [3]

(ii) Calculate the present value at time $t=0$ of a deferred annuity-certain of 5 units per year payable continuously from time $t=4$ to $t=10$. [6]

(iii) Determine, to the nearest 0.1%, the constant annual effective rate of interest earned by an investor who invests the present value calculated in part (ii) at time $t=0$ to obtain the payment stream described in part (ii). [3]

[Total 12]

31 Subject CM1, September 2020, Question 5

A company invests \$50,000 now and receives the following income over the next 12 years:

During the first 4-year period:	\$4,000 per annum paid quarterly in arrears
During the second 4-year period:	\$$X$ per annum paid half-yearly in arrears
During the final 4-year period	\$12,000 per annum paid continuously

There are no other payments undet the investment.

Calculate X assuming the company achieves a nominal rate of return of 9% per annum convertible monthly. [11]

32 Subject CM1, April 2021, Question 4

The force of interest $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t)=\begin{cases} 0.03+0.005t & 0 \le t \le 6 \\ 0.1-0.01t & t > 6 \end{cases}$$

$A(0,t)$, the accumulation at time t of a unit of money invested at time 0, can be written as:

$$A(0,t)=\begin{cases} e^{a+bt+ct^2} & 0 \le t \le 6 \\ e^{f+gt+ht^2} & t > 6 \end{cases}$$

(i) Calculate the values of a, b, c, f, g and h. [5]

A sum of \$5,000 is invested at $t=2$ for 5 years.

(ii) Calculate the annual nominal rate of return convertible monthly on the investment. [3]

[Total 8]

33 Subject CM1, September 2021, Question 8

The force of interest, $\delta(t)$, is a function of time, and at any time, t measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.06 + 0.02t & 0 \le t \le 4 \\ 0.08 - 0.01t & t > 4 \end{cases}$$

$A(0,t)$, the accumulation at time t of a unit of money invested at time 0, can be written as:

$$A(0,t) = \begin{cases} e^{a+bt+ct^2} & 0 \le t \le 4 \\ e^{f+gt+ht^2} & t > 4 \end{cases}$$

(i) Calculate the values of a, b, c, f, g and h. [6]

A sum of \$600 is invested at t = 3 and a further sum of \$900 is invested at t = 9.

(ii) Calculate, showing all working, the accumulated amount at t = 13. [4]

(iii) Calculate, showing all working, the yield of the investment described in part (ii) expressed as an effective rate of interest per month to the nearest 0.1%. [3]

(iv) Comment on your answer to part (iii). [1]

[Total 14]

34 Subject CM1, April 2022, Question 2

An investor is considering making an investment and is deciding between two possible alternatives:

- A 6-month investment which can be purchased at a simple rate of discount of 4.15% *pa*
- A bank deposit, for 6 months, offering an effective rate of interest of 4.35% *pa*

Determine which of these two alternatives offers the higher rate of return. [4]

35 Subject CM1, September 2022, Question 3

An individual aged 20 exact wishes to purchase a commercial property at age 45, which is currently priced at \$1,000,000. They plan to save a fixed amount, X, monthly in advance for the first 10 years, then $2X$ monthly in advance for the remainder of the term.

The investment return is expected to be 2.5% *pa* effective, and property price inflation is expected to be 0.5% effective per half year.

Calculate X , using annuity functions. You must show all working. [5]

36 Subject CM1, September 2023, Question 10 (part)

The force of interest is a function of time and at any time *t* (measured in years) is given by the formula:

$$\delta(t)=\begin{cases}0.03+0.005t & 0\le t<8\\ 0.07 & 8\le t\end{cases}$$

(i) Derive, and simplify as far as possible, expressions for *v*(*t*), where *v*(*t*) is the present value of a unit sum of money due at time *t*.

You should consider separately the cases $0\le t<8$ *and* $t\ge 8$. [5]

(ii) (a) Demonstrate that it will take more than 8 years for an investment to double in value.

(b) Calculate, showing all working, the exact time in years it will take for an investment to double in value. [4]

[Total 9]

37 Subject CM1, April 2024, Question 4

An individual pays £4,000 pa into a savings account for 10 years. During the first 4 years, the payments are made quarterly in advance. For the remaining years, the payments are made continuously.

The investor achieves a yield of 6% *pa* convertible quarterly on the investment.

Calculate, showing all working, the accumulated amount in the savings account at the end of 10 years. [7]

38 Subject CM1, April 2024, Question 10

The force of interest is a function of time and at any time t (measured in years) is given by the formula:

$$\delta(t)=\begin{cases}a+bt & 0\le t<5\\ c & 5\le t\end{cases}$$

where a, b and c are constants.

You are given $v(t)$, the present value of a unit sum of money due at time t:

$$v(t)=\begin{cases} e^{-\left[0.04t+0.01t^2\right]} & 0\le t<5 \\ e^{-\left[0.05t+0.20\right]} & 5\le t \end{cases}$$

(i) Calculate, showing all working, the values of a, b and c. [5]

£1,000 is invested at t = 2 for a period of 7 years.

(ii) Calculate, showing all working, the accumulated value of this investment at t = 9. [3]

(iii) Calculate, showing all working, the rate of interest earned on the investment in part (ii). Express your answer as a percentage rounded to three decimal places as a nominal rate of interest per annum convertible quarterly. [2]

A continuous payment stream is received at a rate of $5e^{0.03t}$ units per annum between t = 8 and t = 13.

(iv) Calculate, showing all working, the present value of the payment stream at t = 0. [4]

[Total 14]

SOLUTIONS TO PAST EXAM QUESTIONS

1 Subject CT1, April 2014, Question 3

(i) ***Nominal rate of interest convertible half-yearly***

We find first the effective annual rate of interest:

$$900(1+i)^{1/3} = 925 \quad \Rightarrow \quad i = 0.085670$$

$$i^{(2)} = 2\left[(1+i)^{1/2} - 1\right] = 2\left[(1.085670)^{1/2} - 1\right] = 8.39\% \ \ pa \text{ (3 SF)}$$

(ii) ***Nominal rate of discount convertible quarterly***

$$d^{(4)} = 4\left[1-(1+i)^{-1/4}\right] = 4\left[1-(1.085670)^{-1/4}\right] = 8.14\% \ \ pa \text{ (3 SF)}$$

(iii) ***Simple rate of interest***

$$900\left(1+\frac{1}{3}i\right) = 925 \quad \Rightarrow \quad i = 8.33\% \ \ pa \text{ (3 SF)}$$

2 Subject CT1, April 2014, Question 11

(i) ***Present value of continuous payment stream***

The value at time 4 of the payment stream is:

$$PV_4 = 3{,}000\left[\bar{a}_{\overline{2}|}^{\delta=7\%} + e^{-2\times 0.07} \times \bar{a}_{\overline{4}|}^{\delta=9\%}\right]$$

Evaluating the functions:

$$\bar{a}_{\overline{2}|}^{\delta=7\%} = \frac{1-e^{-2\delta}}{\delta} = 1.866311$$

$$\bar{a}_{\overline{4}|}^{\delta=9\%} = \frac{1-e^{-4\delta}}{\delta} = 3.359152$$

$$\Rightarrow \quad PV_4 = 3{,}000\left[1.866311 + e^{-2\times 0.07} \times 3.359152\right] = 14{,}359.852$$

We now need to discount this back to time zero. The discount factor from time four to time zero will be:

$$v(4) = e^{-\int_0^4 0.03+0.01s\,ds} = e^{-\left[0.03s+0.005s^2\right]_0^4} = e^{-\left[0.12+0.08\right]} = e^{-0.2}$$

So the amount that would need to be invested at time zero is:

$$PV_0 = e^{-0.2} \times 14{,}359.852 = \$11{,}756.85$$

(ii) ***Equivalent constant rate***

We now need to solve the equation of value for the equivalent constant rate:

$$PV_0 = 3{,}000 v^4 \bar{a}_{\overline{6}|} = 11{,}756.852$$

ie: $$v^4 \times \frac{1-v^6}{\delta} = 3.918951$$

We will use trial and improvement to find a solution to this equation:

$$i = 0.06\,,\ LHS = 4.0107$$

$$i = 0.07\,,\ LHS = 3.7622$$

So, using linear interpolation between these values, we find that:

$$0.06 + \frac{4.0107 - 3.918951}{4.0107 - 3.7622} \times (0.07 - 0.06) = 0.063692$$

So the equivalent constant annual effective rate of interest is 6.4% *pa*.

3 Subject CT1, September 2014, Question 3

(i) ***Annual effective rate of interest***

$$98.83(1+i)^{91/365} = 100 \quad \Rightarrow \quad 1+i = \left(\frac{100}{98.83}\right)^{\frac{365}{91}} \quad \Rightarrow \quad i = 4.834\%\ pa$$

(ii) ***Equivalent simple rate***

$$98.83\left(1+\frac{91i}{365}\right)=100 \quad \Rightarrow \quad i=4.748\%\ pa$$

4 Subject CT1, September 2014, Question 5

(i) ***Monthly annuity***

$$a_{\overline{5}|}^{(12)}=\frac{1-v^5}{i^{(12)}}=\frac{1-1.05^{-5}}{12\left[(1.05)^{1/12}-1\right]}=\frac{1-1.05^{-5}}{0.0488895}=4.42782$$

(ii) ***Deferred annuity***

$${}_{4|}a_{\overline{15}|}=v^4\times a_{\overline{15}|}=1.05^{-4}\times\frac{1-1.05^{-15}}{0.05}=8.53937\ .$$

(iii) ***Increasing annuity paid continuously***

$$\left(I\bar{a}\right)_{\overline{10}|}=\frac{\ddot{a}_{\overline{10}|}-10v^{10}}{\delta}=\frac{\dfrac{1-1.05^{-10}}{1-1.05^{-1}}-10\times1.05^{-10}}{\ln1.05}=40.35012$$

(iv) ***Increasing annuity paid continuously, increasing continuosly***

$$\left(\bar{I}\bar{a}\right)_{\overline{10}|}=\frac{\bar{a}_{\overline{10}|}-10v^{10}}{\delta}=\frac{\dfrac{1-1.05^{-10}}{\ln1.05}-10\times1.05^{-10}}{\ln1.05}=36.36135$$